Fluid Flow in Compressible Porous Media: I: Steady-State Conditions

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This is the first of two articles dealing with fluid flow in compressible porous media. In this article a model describing fluid flow and pressure-induced variations in porosity under stationary conditions is developed. In a forthcoming article the dynamic behavior during filtration and wet pressing of compressible porous media are presented.

Fluid flow through rigid porous media is generally described by Darcy's law. The corresponding expression for compressible materials is derived in this article. This expression, the steady-state flow (SSF) equation, allows the steady-state flow to be easily calculated, either numerically, or by using the approximative analytical solutions that are also presented here. In the SSF equation optional empirical and/or theoretical permeability and compressibility relationships may be combined. Further, a new compressibility model which also applies for viscoelastic materials is presented. The influence of the compressibility of the material and the influence of precompression is illustrated.

Introduction

The description of such diversified applications as ground water hydrology, petroleum engineering, filtration in gel filter media, washing of filter cakes and wet pressing of paper, for example, relies on the physics of flow in porous media. Hence, the aspects of flow in porous media have been discussed in a vast number of articles. Comprehensive reviews of this subject have been published by Muskat (1937), Scheidegger (1957), Happel and Brenner (1965), Philip (1970) and Dullien (1979).

Most articles describe fluid flow in rigid materials, or materials that are regarded as being rigid. The significant properties of these media are the porosity and the permeability. The porosity is a measure of the pore space, and hence of the fluid capacity of the medium, while the permeability is a measure of the ease with which fluids may traverse the medium under the influence of an applied pressure.

Fluid flow in compressible materials is influenced by a third property, the compressibility. The compressibility is a measure of the mechanical strength of the material. Compressible materials, such as gel media and fibrous materials, present a unique problem in attempts to relate fluid flow and applied pressure, mainly because of a variation in bed porosity throughout the bed during filtration.

The reduction in porosity of a compressible material subjected to drag forces of a fluid or to a mechanical pressure is due to any, or all, of the following (Gurnham and Mason, 1946): deformation of the solid matter, bending and slipping of individual particles and disintegration of solid particles. The porosity reduction mechanisms are illustrated in Figure 1.

The theory of laminar flow through homogeneous porous media is based on a classical experiment originally performed by Darcy (1856). In Darcy's law it is stated that the volume rate of flow, Q, through a bed of cross-sectional area A is correlated to the frictional pressure drop, ΔP, across the bed:

$$Q=\frac{K_D A \Delta P}{\mu \Delta L} \quad (1)$$

where K_D is the specific permeability of the porous medium, μ is the fluid viscosity and ΔL is the thickness of the medium. The permeability K_D is characterized by the specific properties of the porous medium; properties such as the porosity, the shape and orientation of the solid particles, the surface area exposed to the fluid, and the pore-size distribution.

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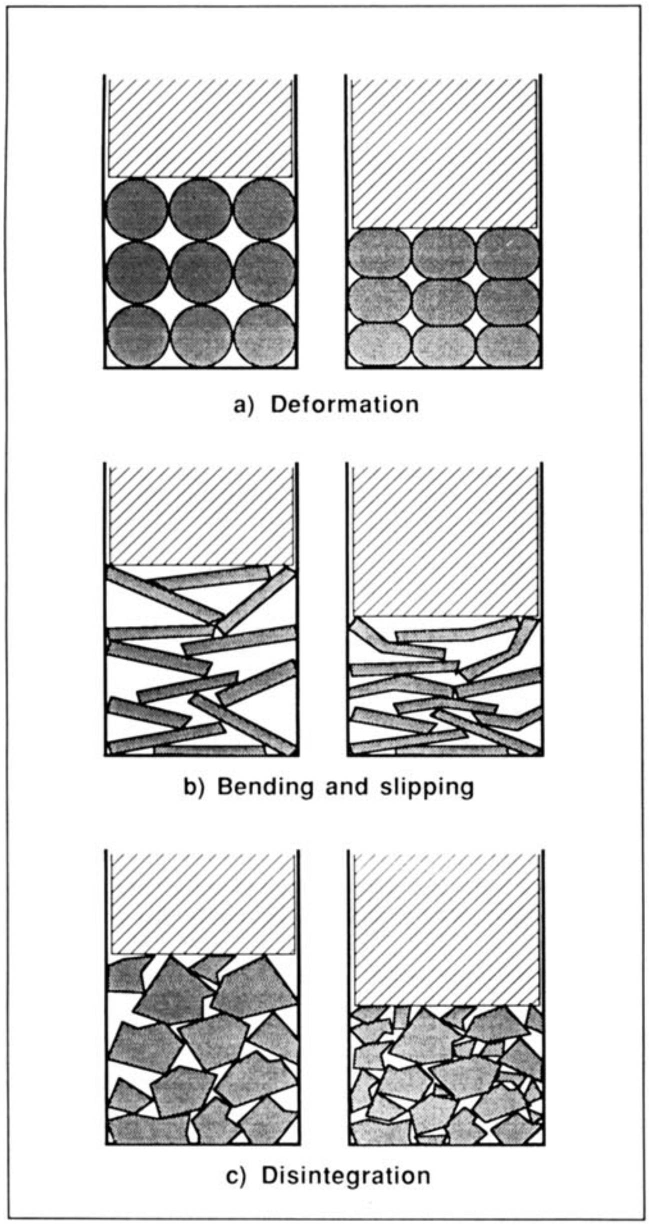

Figure 1. Mechanisms of pore space reduction induced by compression of granular and fibrous materials.

Fluid flow through porous media is accurately described by Darcy's law only for rigid materials. A corresponding expression for compressible materials is derived in this article. This steady-state flow equation (the SSF equation) describes how the porosity and fluid flow through a compressible medium are influenced by hydraulic and/or mechanical pressure. Empirical, as well as theoretical, permeability and compressibility relationships can be combined in the SSF equation.

A great number of permeability relationships for different types of materials have been presented; some of the most commonly used are reviewed in this article. There are, however, few compressibility relationships available and especially the lack of a relationship that accurately describes the behavior of viscoelastic materials, such as gel media and fiber beds, has limited the application of existing compressibility relationships. The typical hysteresis exhibited by the compression-recovery cycle of viscoelastic material is clearly illustrated in Figure 2.

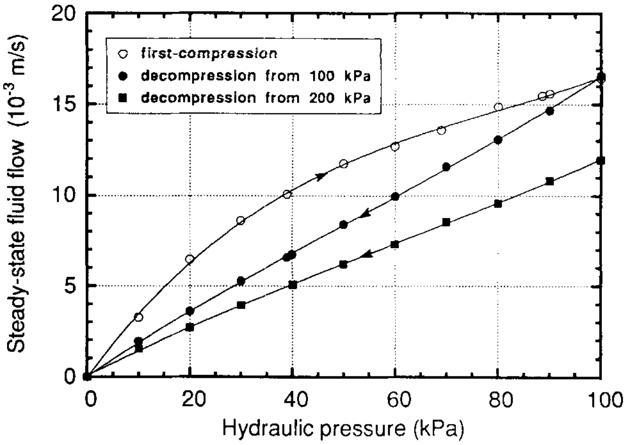

Figure 2. Compression-recovery cycle of a bed of cotton-wool fibers with a basis weight of 30 kg/m^2.

The new compressibility model, presented in this article, is applicable in a wider pressure range than previously available compressibility relationships, and also describes the behavior of precompressed materials, exemplified in Figure 2.

Two examples of the application of the model are given; the influence of the mechanical properties of compressible media is presented and the influence of precompression is illustrated.

Steady-State Flow Equation for Compressible Media

As stated earlier, the porosity, ϵ, is the commonly used measure of the fraction of a porous medium available for fluid flow. The porosity is defined as the ratio between the void volume, V_v, and the total volume of the medium, V_{tot}:

$$\epsilon = \frac{V_v}{V_{tot}} = 1 - \frac{V_{sa}}{V_{tot}} \tag{2}$$

where V_{sa} is the volume of solid material and adsorbed water.

However, in this article the derivation of the steady-state flow equation is based on the void ratio, X, defined as:

$$X = \frac{V_v}{V_{sa}} \tag{3}$$

The use of the void ratio, instead of porosity, in the derivation of the SSF equation is justified by the simplicity of the compressibility and permeability equations when expressed as a void ratio. This significantly facilitates the subsequent numerical calculations. However, the initial and resulting values may be given as porosity or solids content if desired. The use of the void ratio may thus be restricted to the numerical calculations.

The relation between porosity and void ratio is:

$$\epsilon=\frac{X}{1+X} \tag{4}$$

and between total dry solids content (TDS) and void ratio:

$$\mathrm{TDS}=\frac{m_s}{m_s+m_w}=\frac{v_w}{Xv_{sa}+v_w(1+m_a/m_s)} \tag{5}$$

where m_s, m_a and m_w are the amounts of solids, adsorbed water and the total amount of water in the sample, respectively. The ratio between the amount of absorbed water and amount of solids, m_a/m_s, is the adsorption ratio, v_w is the specific volume of water, and v_{sa} is the specific obstruction volume defined in Eq. 6.

Adsorbed water

The incompressible volume, V_{sa}, is equal to the volume of the solid material, V_s, for nonswelling solids, for example, glass fibers. For other materials, for example, gel media and cellulose fibers, the volume of the adsorbed, immobilized water constitutes a considerable part of the incompressible volume. For cellulose fiber beds the adsorption ratio is approximately 0.3 grams of water per gram of dry cellulose (Campbell, 1947), for example.

The volume occupied by the solid material and adsorbed water, V_{sa}, is in this article regarded as an incompressible part of the medium; that is the volume of solid matter and adsorbed water in the medium is constant, and the deformation, bending, slipping and disintegration of the solid particles affect only the void volume, V_v.

The volume of the incompressible fraction of the medium may be expressed as:

$$V_{sa}=m_s v_{sa} \tag{6}$$

The specific obstruction volume, v_{sa}, is related to the specific volume of sc' ls, v_s, and water, v_w, by the relation:

$$v_{sa}=v_s+\frac{m_a}{m_s}v_w \tag{7}$$

A new coordinate system

A compressible medium constitutes a complex multicomponent system with mutual transport of water and solid material. This makes it difficult to describe fluid flow in compressible media theoretically. In this work this problem is simplified by the introduction of the approximation that there is no mixing of solid material between different layers in the solid matrix. A layer may be compressed, or expanded, but no transport of solid material out from, or into, the layer is allowed. This is probably a good appı əximation for fibrous materials, especially at high concentrations, although mixing almost certainly occurs at lower concentrations, especially for granular materials.

The assumption of there being no inter-layer transport of solid material makes it practicable to introduce a new coordinate system which deforms with the medium. This coordinate system is denoted the y-coordinate system in this article. The y coordinate for a solid particle, or a domain of adsorbed water, in this new coordinate system is independent of time and applied pressure since the coordinate system is assumed to deform with the medium. As there is no interlayer transport of solid material the volume of the solid material and adsorbed water, ΔV_{sa}, in a volume element of thickness Δy is unchanged during compression; that is,

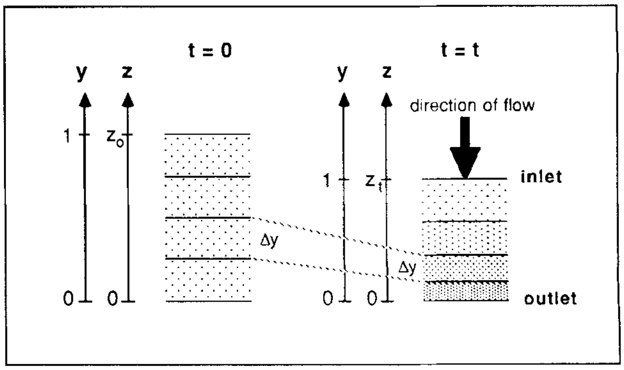

Figure 3. Schematic representation of the *z*- and *y*-coordinate systems.

The z-coordinate system is a normal system, and the y-coordinate system is a system in which the amount of solid material in a volume element of thickness Δy is constant during compression.

$$\frac{dV_{sa}}{dy}=\text{constant} \tag{8}$$

In Figure 3 the two coordinate systems are visualized: the real z-coordinate system and the imaginary y-coordinate system.

The y-coordinate system is by definition normalized to $0 \le y \le 1$ and the amount of solid material in each Δy element is also, by definition, constant:

$$\Delta m_s=WA\,\Delta y \tag{9}$$

where W is the basis weight, that is, the amount of the solid component per unit area.

The volume of the incompressible part of the medium is:

$$\Delta V_{sa}=\Delta(m_s v_{sa})=\frac{1}{1+X}A\,\Delta z \tag{10}$$

The correlation between the z- and y-coordinate system is obtained by combining Eqs. 9 and 10:

$$\frac{dz}{dy}=Wv_{sa}(1+X) \tag{11}$$

The flow equation

When a compressible medium is subjected to hydraulic and/or mechanical pressure the total volume of the medium is decreased while at the same time fluid flows through the medium. The total pressure, P_{tot}, may be divided into two separate pressure components; the mechanical stress exerted on the medium, P_m, and the hydraulic pressure which gives rise to the fluid flow, P_h. Both these pressure components vary with position in the medium, but their sum is constant and equal to the total pressure throughout the medium:

$$P_{\text{tot}} = P_m + P_h \tag{12}$$

The relation between the fluid flow per cross-sectional area of the medium, J, and the pressure drop across an element of thickness Δz is expressed by the differential form of Darcy's law:

$$J = \frac{K_D}{\mu} \frac{dP_h}{dz} \tag{13}$$

In this work, J, is defined as being positive in the negative z direction (see Figure 3).

In order to simplify the subsequent numerical calculations a dimensionless permeability function is introduced. The new permeability function, $K(X)$ is defined as:

$$K(X) = \frac{K_D}{1+X} \left(\frac{S_s}{v_{sa}}\right)^2 \tag{14}$$

where S_s is the surface exposed to the fluid per unit mass of solid material. The specific surface, S_w, used in most permeability expressions is defined as the surface exposed to the fluid per unit volume of solid material, or for a material containing adsorbed water:

$$S_w = \frac{S_s}{v_{sa}} \tag{15}$$

Equations 11 and 14 allow Eq. 13 to be rewritten as:

$$J(y) = \frac{v_{sa}}{\mu W S_s^2} K(X) \frac{dP_h}{dy} \tag{16}$$

The derivative of the hydraulic pressure with respect to y is:

$$\frac{dP_h}{dy} = \frac{d(P_{\text{tot}} - P_m)}{dy} = -\frac{dP_m}{dy} \tag{17}$$

At steady state the flow $J(y)$ is constant throughout the medium, and is hence independent of the position, y, in the medium. The following equation is then obtained from Eqs. 16 and 17:

$$\int_0^y J(y)dy = J_{\text{SSF}}\, y = \frac{v_{sa}}{\mu W S_s^2} \int_{P_m(y)}^{P_m+P_h} K[X(P)]dP \tag{18}$$

where J_{SSF} is the steady-state flow, $P_m(y)$ is the mechanical pressure exerted on the matrix at y and $P_m + P_h$ is the mechanical pressure at the position where water leaves, that is, at $y = 0$.

The SSF equation, Eq. 18, together with expressions for the compressibility [$X(P)$ in the above equation] and the permeability, function $K(X)$, permits the derivation of the steady-state flow J_{SSF}. Before application of the SSF equation some compressibility and permeability correlations suitable for introduction into Eq. 18 will be presented; a general compressibility model is derived and a number of commonly used permeability relationships are reviewed.

Compressibility

The volume reduction of a porous medium subjected to a pressure is due to deformation of the solid matter, bending and slipping of individual particles and/or disintegration of solid particles. Bending, slipping and disintegration of the solid material are essentially irreversible processes. Hence, media in which these deformation mechanisms are of importance are subject to nonrecoverable volume reductions. The amount of nonrecoverable deformation is especially pronounced during the first compression, and then decreases as deformation is repeated (Wilder, 1960; Han, 1969).

Some empirical relationships between volume reduction and applied pressure are available. However, no fundamental theoretical compressibility correlation has yet, at least to the authors' knowledge, been published.

In this chapter a semiempirical compressibility model is presented, which is applicable in a wider pressure range than previously available empirical relationships. The presented model also has the advantage of describing the volume recovery of porous media. This feature is especially useful when considering materials that are repeatedly compressed and expanded, for example, fiber mats in paper machines with several press roll nips.

Basic compressibility concept

Compressibility is a measure of the volume change of a system subjected to a compressive stress. The compressibility, β, is defined as the rate of variation of volume with pressure at a constant temperature (Moore, 1972):

$$\beta = -\frac{1}{V} \left(\frac{\partial V}{\partial P_m}\right)_T \tag{19}$$

For a wide variety of materials it has been found that the correlation of the physical compressibility to applied pressure, during the first compression of the material, follows the following relationship (Grace, 1953):

$$\beta = N P_m^{-b} \tag{20}$$

where N and b are material-specific parameters. The value of N varies widely for different materials, whereas $b \approx 1$ for most materials examined. The compressibility of various materials can be found in Figure 4.

When $b = 1$ is assumed the following relationship is obtained (Jönsson, 1983):

$$C = M P_m^N \tag{21}$$

Equation 21, which was first suggested by Qviller (1938) and later by Campbell (1947), has been used extensively to correlate concentration C and pressure P_m for fibrous materials.

Though successfully applied in low-pressure regions, Eqs. 20 and 21 are not universally applicable as they do not predict a finite value for the concentration at high pressures. We have

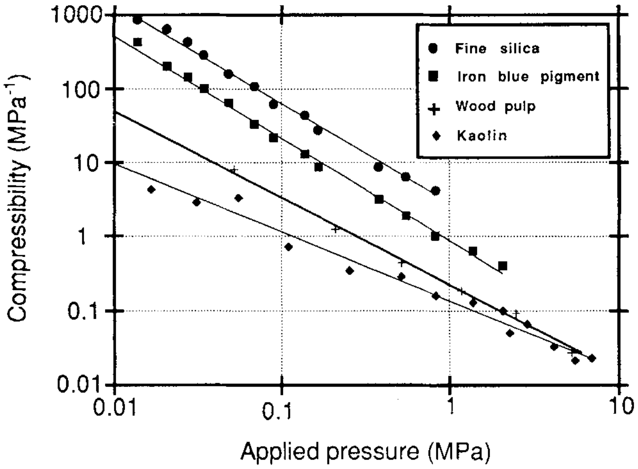

Figure 4. Relation of compressive pressure to cake compressibility.

Data for the nonfibrous materials from Grace (1953) and for the wood pulp from Turner et al. (1948).

reduced this shortcoming by relating the compressibility to the void volume instead of the total volume, hence:

$$\beta_v = NP_m^{-b} \tag{22}$$

where β_v is the compressibility of the void volume.

The basis of Eq. 22 is the assumption that porous media may be divided into a number of immiscible volumes. The total volume of the medium can be divided into the volume of solid material, V_s, the volume of adsorbed water, V_a, and the void volume, V_v:

$$V_{tot} = \sum_i V_i = V_s + V_a + V_v \tag{23}$$

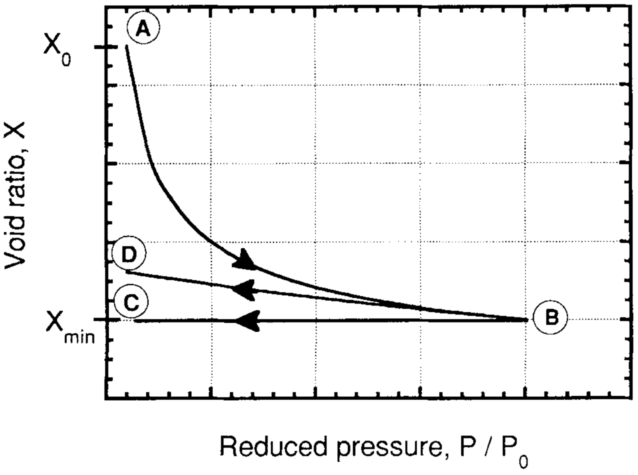

Figure 5. At the beginning of the compression, at point A, the void ratio is X_o. During the compression phase, to point B, the volume reduction is related to the applied pressure according to Eq. 24.

In the extreme cases, a totally elastic and a nonelastic material, the void ratio is X_o (point A) and X_{min} (point C), respectively, when the compressive stress is removed. The curve B-D shows the void ratio recovery of a viscoelastic material.

where each volume has a specific compressibility β_i. The compressibility of solid particles and adsorbed water is generally very small, while the volume of the interstices is sensitive to a change in the compressive pressure. The volume of solid material and adsorbed water is therefore considered to be incompressible, and hence $\beta_s = 0$ and $\beta_a = 0$.

When the compressibility is related exclusively to the void volume, a combination of Eq. 19 and Eq. 22, with $b = 1$, yields:

$$X = X'\left(\frac{P_m}{P'_m}\right)^{-N} \tag{24}$$

where X' is the void ratio at pressure P'_m for the first compression of the medium.

Compressibility after the first compression

The compression-recovery cycle of materials such as gel media and fiber beds exhibits hysteresis. The degree of recovery depends on the physical properties of the medium and the applied pressure. The initial volume of an elastic material is completely recovered when the compressive stress is removed, whereas for a viscoelastic material a certain nonrecoverable deformation remains. Compression-recovery cycles for different types of materials are shown in Figure 5.

The amount of nonrecoverable deformation decreases as the deformation is repeated until a point is reached at which no more nonrecoverable deformation is observed. Such a material is said to be mechanically conditioned.

The influence of pressure on the compressibility after the first compression may be expressed either by assigning a lower value to N in Eq. 22, or by neglecting the pressure dependence of β after the first compression. In this article we have chosen to use the second alternative, that is, assuming the compressibility to be constant after the first compression. It should, however, be pointed out that this implies no restrictions on the use of the SSF equation. Whichever compressibility relation is applicable, it can be used in the SSF equation.

In this article we have thus chosen to treat the compressibility during the first compression as an irreversible process, described by Eq. 22, whereas after the first compression the material is treated as being elastic with constant compressibility:

$$\beta_v = NP_{max}^{-1} \tag{25}$$

where P_{max} is the highest compressive stress the material has been exposed to. At pressures below the maximum applied pressure the equation corresponding to Eq. 24 is derived by the combination of Eqs. 25 and 19:

$$X = X(P_{max})\exp\left(N - \frac{NP_m}{P_{max}}\right) \tag{26}$$

Permeability

Comprehensive reviews of various permeability relationships have been given by, for example, Scheidegger (1957), Dullien (1979), and Jackson and James (1986). Below, some of the most commonly used correlations are briefly reviewed.

Empirical relationships

If there already exists an empirical relation for the medium in question, then this relation between permeability and void ratio should, of course, be used. For limited porosity intervals relatively simple correlations are often obtained. For example, the permeability of different paper pulps has been found to follow the exponential equation (Carlsson et al., 1983):

$$K_D = q_1 MR^r \approx qX^r \tag{27}$$

where MR is the moisture ratio, kg water/kg dry pulp, and q and r are constants specific to different types of paper pulps.

Correlations applicable in a wider porosity interval must, however, be based on a theoretical model.

Theoretical relationships

The most widely used of the expressions relating the classical fluid flow concept of Darcy to some of the particle properties is the Kozeny-Carman equation where the specific permeability is evaluated as (Scheidegger, 1957):

$$K_D = \frac{\epsilon^3}{kS_w^2\,(1-\epsilon)^2} \tag{28}$$

where k is the so-called Kozeny constant. The Kozeny-Carman equation may be transformed into the form of the permeability function used in this paper by the insertion of Eqs. 4, 14 and 15 into Eq. 28:

$$K(X) = \frac{X^3}{k(1+X)^2} \tag{29}$$

Carman (1937) has shown that for beds composed of sand and randomly packed powders $k \approx 5.0$. For flow through beds of glass spheres k were found to be 4.50 (Sullivan and Hertel, 1940) and 4.65 (Musket and Botset, 1931).

For beds composed of fibers the Kozeny constant is influenced by both porosity and the orientation of the fibers. The Kozeny-Carman equation has been found to apply to randomly packed fibrous beds in the porosity range from 0.55 to 0.86 with an average value of 5.55 (Fowler and Hertel, 1940). At high porosities, above 0.86, the Kozeny constant was found to be strongly dependent on the porosity (Sullivan, 1941; Ingmanson et al., 1959).

The Kozeny constant is dependent on the orientation of the fibers, and was found to be 3.07 for flow through a bed of glass fibers oriented approximately parallel, and 6.04 for fibers oriented perpendicular, to the direction of flow (Sullivan and Hertel, 1940).

In spite of the weaknesses of the Kozeny-Carman equation, it has been widely used to relate the fluid flow through fiber beds to fiber properties, notably specific surface and specific volume (Robertson and Mason, 1949; Ingmanson, 1952; Ingmanson and Whitney, 1954; Thode and Ingmanson, 1959; Robertson, 1963).

The Kozeny-Carman equation was originally derived for flow through cylindrical channels. For flow around fibers the Happel equation is preferable. One of the most successful permeability expressions for fibrous materials (Meyer, 1962; Han, 1969) appears to be Happel's free-surface model (Happel, 1959; Happel and Brenner, 1965). Happel's model is based on the assumption that the bed consists of a regular assemblage of cylinders. Each cylinder is assumed to be surrounded by a concentric envelope of fluid. For the case of flow perpendicular to the cylinders the permeability becomes:

$$K(X) = -0.5 + 0.5\,\ln\,(1+X) + \frac{1}{(2+2X+X^2)} \tag{30}$$

and for the case of flow parallel to the cylinders, the permeability function $K(X)$ was found to be:

$$K(X) = \ln\,(1+X) - \frac{X(1+1.5\,X)}{(1+X)^2} \tag{31}$$

The specific permeability of a bed composed of spheres has also been derived on the basis of a cell model (Happel, 1958). The permeability was evaluated as:

$$K(X) = 2 - \frac{3}{(1+X)^{1/3}} + \frac{5}{3(1+X)^{5/3}+2} \tag{32}$$

The above theoretical permeability equations were derived for a homogeneous bed. The case of flow through beds with randomly oriented fibers and nonuniform cell size has been treated elsewhere (Jönsson, 1983), but is beyond the scope of this article.

Application of the Model

When the general relationships for $X(P)$ and $K(X)$ are used, the SSF equation must usually be solved numerically. This is, however, easily performed with standard programs on a PC. Most of the examples treated in this article are solved numerically, for example. However, in order to illustrate the use of the SSF equation an approximate solution is presented below.

An approximate solution of the SSF equation

The derivation of an approximate solution of the SSF equation is suitably based on the fact that the volume flow is primarily controlled by the void ratio in the compact layer where water leaves the medium, $X_{y=0}$. The permeability function, $K(X)$, may be approximated by the first two terms in a Taylor series expansion:

$$K(X) = K(X_{y=0})\left[1 + \alpha\,\ln\left(\frac{X}{X_{y=0}}\right)\right] \tag{33}$$

where α is a parameter used to provide the derivative of the function at $X_{y=0}$.

$K(X_{y=0})$ and α are easily experimentally determined parameters; α is the slope of the curve when $K(X)$ is plotted as a function of X in a logarithmic diagram. The expression for α is:

$$\alpha = \left[\frac{d[\ln\,(K(X))]}{d[\ln\,(X)]}\right]_{X_{y=0}} \tag{34}$$

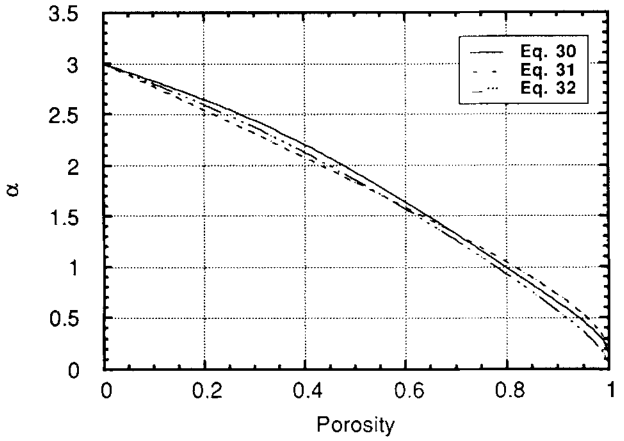

Figure 6. The parameter α evaluated for flow perpendicular to cylinders, Eq. 30, flow parallel to cylinders, Eq. 31, and flow around spheres, Eq. 32.

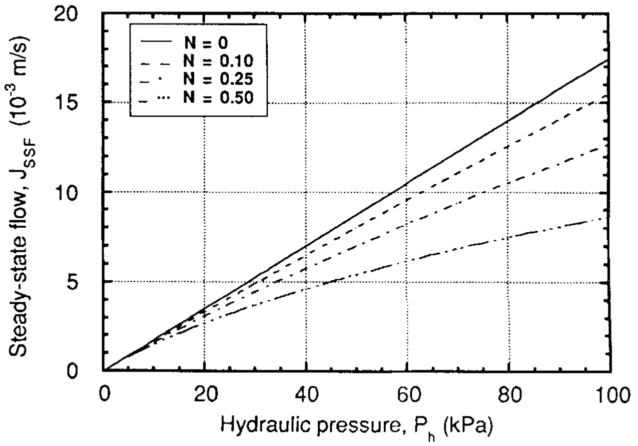

Figure 7. Influence of compressibility on fluid flow at varying applied hydraulic pressure.

The parameters in the SSF equation were assigned the following values: $v_{sa} = 0.92 \times 10^{-3}$ m³/kg; $\mu = 0.9 \times 10^{-3}$ Pa·s; $S_s = 2{,}000$ m²/kg; and $W = 1$ kg/m². A constant applied mechanical pressure of 8.13 kPa, corresponding to an initial porosity of 0.90 (= solids content of 10%) was also used. For cellulose fiber beds $N = 0.3$–0.5 (Han, 1969).

In Figure 6 the value of α is evaluated for flow through beds of spheres and flow perpendicular and parallel to cylinders. As shown in Figure 6, α is only slightly affected by the shape and orientation of the particles. The value of α is about 3 when $\epsilon = 0$ and ≈ 0 when $\epsilon = 1$.

At the outlet of the medium, where $X = X_{y=0}$, the mechanical stress exerted on the solid matrix is the sum of the applied mechanical and hydraulic pressures, $P_{m,y=0} = P_m + P_h$. For the first compression of a medium, the relation between the permeability and the applied pressure is obtained by the insertion of Eq. 24 into Eq. 33.

$$K(X(P)) = K(X_{y=0})\left[1 - \alpha N \ln\left(\frac{P}{P_m + P_h}\right)\right] \tag{35}$$

The steady-state flow through the bed, J_{SSF}, may then be calculated from Eq. 18.

$$J_{\mathrm{SSF}} = \frac{v_{sa}}{\mu W S_s^2}\int_{P_m}^{P_m+P_h} K(X_{y=0})\left[1 - \alpha N \ln\left(\frac{P}{P_m + P_h}\right)\right] dP$$
$$= \frac{v_{sa}}{\mu W S_s^2} K(X_{y=0})\left[(1+\alpha N)P_h + \alpha N P_m \ln\left(\frac{P_m}{P_m + P_h}\right)\right] \tag{36}$$

Equation 36 may be further simplified by using a series expansion of the logarithmic expression. The errors introduced by this simplification are relatively small.

$$J_{\mathrm{SSF}} = \frac{v_{sa}}{\mu W S_s^2} K(X_{y=0}) P_h \left(1 + \alpha N \frac{P_h}{2P_m + P_h}\right) \tag{37}$$

As has been demonstrated earlier in this article the compressibility of a medium is different for the first compression compared with the following compressions. If instead of Eq. 24, Eq. 26 is used to obtain the relationship between mechanical stress and equilibrium void ratio the following equation for the steady-state flow is obtained:

$$J_{\mathrm{SSF}} = \frac{v_{sa}}{\mu W S_s^2} K(X_{y=0}) P_h \left(1 + \alpha N \frac{P_h}{2P_{\max}}\right) \tag{38}$$

The first part of Eqs. 37 and 38 can be recognized as Darcy's law, Eq. 1, for a medium with a void ratio $X_{y=0}$.

In cases where an approximate analytical solution is not sufficiently accurate, the SSF equation should be solved numerically. Below, the influence of various parameters is derived numerically. In these examples it is assumed that the compressibility and permeability of the porous media are described by Eqs. 24, 26 and 30.

Influence of the mechanical properties of the compressible media

The parameter N, introduced into the compressibility Eq. 20, is a measure of the mechanical strength of the material. N is equal to zero for a rigid material, and in this case the influence of pressure on fluid flow follows Darcy's law. As the value of N increases, that is, as the ability to sustain a mechanical stress decreases, the fluid flow decreases, as shown in Figure 7.

The lower permeability of a more compressible medium is due to the steeper porosity gradient in such a medium. As shown in Figure 8, the porosity is the same at the inlet, that is, at $y = 1$, independent of the mechanical strength of the material, whereas the porosity at the outlet, that is, at $y = 0$, is highly susceptible to the mechanical properties of the material. The lower fluid flow through a compressible material is thus due mainly to the lower porosity in the bottom layer of the medium.

Fluid flow through precompressed materials

The fluid flow is higher during the first compression of a material than through materials that have been precompressed. The greater the compressive stress, $P_{\max}$, the material has been subjected to, the lower the fluid flow, as shown in Figure 9.

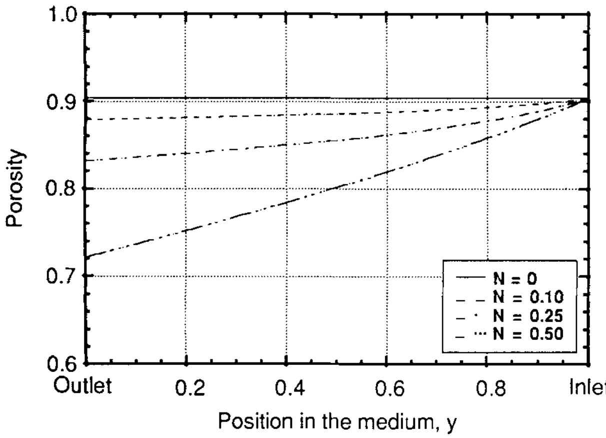

Figure 8. Porosity gradient in materials of differing mechanical strength at an applied hydraulic pressure of 100 kPa.

Values of the compressibility and permeability parameters are the same as in Figure 7.

Conclusion

Darcy's law, commonly used to describe the physics of flow in porous media, fails to predict the relation between fluid flow and pressure drop in compressible media. In this article a fluid flow equation is presented which makes it possible to calculate the resistance to flow of compressible porous media. Both fluid flow and variation of porosity throughout, for example, gel filter media and fiber beds, are easily derived from this equation.

The information necessary to solve the presented steady-state flow equation is static data on the compressive properties, the compressibility, of the medium, and flow resistance, permeability, as a function of porosity.

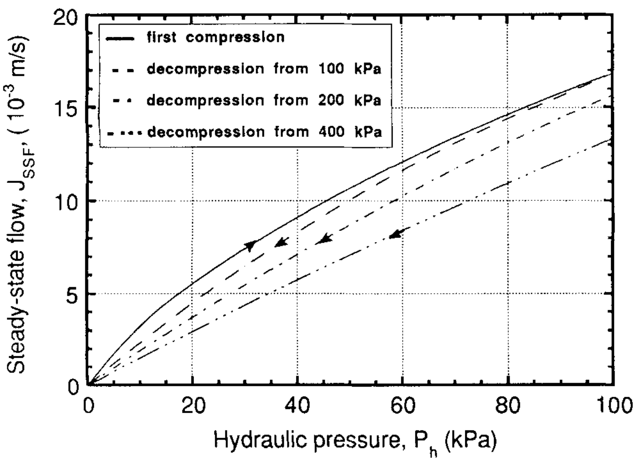

Figure 9. Fluid flow during first compression of a medium and fluid flow through materials that previously have been subjected to a compressive stress, P_{max}.

The parameters in the SSF equation were assigned the following values: $v_{sa} = 0.92\times10^{-3}$ m^3/kg; $\mu = 0.9\times10^{-3}$ Pa·s; $S_s = 290$ m^2/kg; $W = 30$ kg/m^2; and $N = 0.45$. A constant applied mechanical pressure of 8.13 kPa, corresponding to an initial porosity of 0.90 (= solids content of 10%) was also used.

Approximate analytical solutions for flow through first-compression and precompressed media are given. The flow/pressure drop values derived from the approximate analytical solutions are accurate enough for most practical purposes. It is thus usually not necessary to solve the SSF equation numerically. This is especially true for media where there is a lack of precise knowledge concerning the compressibility and permeability properties of the medium.

It is demonstrated how fluid flow decreases as the ability of the medium to sustain mechanical stress decreases. Precompression of viscoelastic media has a definite influence on the fluid flow, which is also demonstrated.

A slow decline in flow with time is sometimes observed when fluid flow in compressible porous media is studied experimentally. The origin of this decline is a gradual decrease in the porosity due to blocking of the flow paths by colloids and fine particles. This is a phenomenon not encountered in the flow model presented in this article. For materials with a high proportion of colloids the blocking effect dominates the flow behavior of the material. However, if the colloid fraction is reduced, for example, by the addition of polyelectrolytes, the flow behavior of the material becomes similar to that predicted by the SSF model (Jönsson, 1987).

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Notation

A = cross-sectional area
b = material-specific compressibility parameter
C = concentration
J = fluid flow per cross-sectional area
J_{SSF} = steady-state fluid flow per cross-sectional area
K = permeability
K_D = specific permeability
k = Kozeny constant
M = material-specific compressibility parameter
MR = moisture ratio
m_a = amount of adsorbed water
m_s = amount of solids
m_w = total amount of water
N = material-specific compressibility parameter
P_h = hydraulic pressure of the fluid
P_m = mechanical stress exerted on the medium
P_{max} = highest pressure the material has been exposed to
P_{tot} = total pressure
Q = volume rate of flow
q = material-specific permeability parameter
r = material-specific permeability parameter
S_s = surface exposed to the fluid per unit mass of solids
S_w = surface exposed to the fluid per unit volume of solids
TDS = total dry solids content
V_a = volume of adsorbed water
V_s = volume of solid material
V_{sa} = volume of solid material and adsorbed water
V_{tot} = total volume
V_v = void volume
W = amount of solid component per unit area
X = void ratio
y = length coordinate in an imaginary coordinate system
z = length coordinate in a normal coordinate system

Greek letters

α = slope of the curve ln $K(X)$ as a function of ln X
β = compressibility

ΔP = pressure drop across the medium
ΔL = thickness of the medium
ϵ = porosity
μ = fluid viscosity
v_s = specific volume of solids
v_{sa} = specific obstruction volume
v_w = specific volume of water

Literature Cited

Campbell, W. B., "The Physics of Water Removal," *Pulp Paper Mag. Can.*, **48**(3), 103 (1947).

Carlsson, G., T. Lindström, and T. Florén, "Permeability to Water of Compressed Pulp Fiber Mats," *Sv. Papperstidn.*, **86**, R128 (1983).

Carman, P. C., "Fluid Flow Through Granular Beds," *Trans. Inst. Chem. Eng.*, **15**, 150 (1937).

Darcy, H. P. G., "Les Fontaines Publiques de la Ville de Dijon," Victor Dalmont, Paris (1856).

Dullien, F. A. F., "Porous Media—Fluid Transport and Pore Structure," Academic Press (1979).

Fowler, J. L., and K. L. Hertel, "Flow of Gas Through Porous Media," *J. Appl. Physics*, **11**, 496 (1940).

Grace, H. P., "Resistance and Compressibility of Filter Cakes," *Chem. Eng. Progr.*, **49**(6), 303 (1953).

Gurnham, C. F., and H. J. Masson, "Expression of Liquids from Fibrous Materials," *Ind. Eng. Chem.*, **38**(12), 1309 (1946).

Han, S. T., "Compressibility and Permeability of Fiber Mats," *Pulp Paper Mag. Can.*, **70**, T134 (1969).

Happel, J., "Viscous Flow in Multiparticle Systems: Slow Motion of Fluids Relative to Beds of Spherical Particles," *AIChE J.*, 4(2), 197 (1958).

Happel, J., "Viscous Flow Relative to Arrays of Cylinders," *AIChE J.*, 5(2), 174 (1959).

Happel, J., and H. Brenner, "Low Reynolds Number Hydrodynamics," Prentice Hall Inc. (1965).

Ingmanson, W. L., "An Investigation of the Mechanism of Water Removal from Pulp Slurries," *Tappi*, **35**(10), 439 (1952).

Ingmanson, W. L., and R. P. Whitney, "The Filtration Resistance of Pulp Slurries," *Tappi*, **37**(11), 523 (1954).

Ingmanson, W. L., B. D. Andrews, and R. C. Johnson, "Internal Pressure Distributions in Compressible Mats under Fluid Stress," *Tappi*, **42**(10), 840 (1959).

Jackson, G. W., and D. F. James, "The Permeability of Fibrous Porous Media," *Can. J. Chem. Eng.*, **64**, 364 (1986).

Jönsson, A.-S., "A New Method for the Estimation of Residual Lignin Content in Paper Pulp," PhD Thesis, Dep. of Chemical Engineering I, Lund University, Lund (1983).

Jönsson, B., E. Petersson, and B. Lindman, "Mechanical Dewatering of Peat," *Fuel*, **66**, 785 (1987).

Meyer, H., "A Filtration Theory for Compressible Fibrous Beds Formed from Dilute Suspensions," *Tappi*, **45**(4), 296 (1962).

Moore, W. J., "Physical Chemistry," Prentice-Hall Inc. (1972).

Muskat, M., and H. G. Botset, "Flow of Gas through Porous Materials," *Physics*, **1**, 24 (1931).

Muskat, M., "The Flow of Homogeneous Fluids through Porous Media," McGraw-Hill Book Company (1937).

Philip, J. R., "Flow in Porous Media," *Ann. Rev. of Fluid Mech.*, Vol. 2 (1970).

Qviller, O., "Utpressning av Vann av Cellulose," *Papir-J.*, No 23, 312 (1938).

Robertson, A. A., and S. G. Mason, "Specific Surface of Cellulose Fibers by the Liquid Permeability Method," *Pulp Paper Mag. Can.*, **50**(13), 103 (1949).

Robertson, A. A., "The Physical Properties of Wet Webs," *Sv. Papperstidn.*, **66**(12), 477 (1963).

Scheidegger, A. E., "The Physics of Flow through Porous Media," Univ. of Toronto Press (1957).

Sullivan, R. R., and K. L. Hertel, "The Flow of Air through Porous Media," *J. Appl. Phy.*, **11**, 761 (1940).

Sullivan, R. R., "Further Study of the Flow of Air through Porous Media," *J. Appl. Phy.*, **12**, 503 (1941).

Thode, E. F., and W. L. Ingmanson, "Factors Contributing to the Strength of a Sheet of Paper: I. External Specific Surface and Swollen Specific Volume," *Tappi*, **42**(1), 74 (1959).

Turner, H. D., J. P. Hohf, and S. L. Schwartz, "Effect of Some Manufacturing Variables on the Properties of Fiberboard Prepared from Milled Douglas-Fir," *Paper Trade J.*, **127**(6), 43 (1948).

Wilder, H. D., "The Compression and Creep Properties of Wet Pulp Mats," *Tappi*, **43**(8), 715 (1960).

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